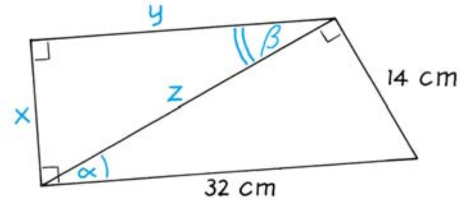
**Opgaven 4.1 – Scalars en vectoren**

| | | | |
|---|---|---|---|
| **0** | ***a*** | $\sin\alpha = 0{,}33 \Rightarrow \alpha = 19^\circ$ | 19° |
| | ***b*** | $\tan\alpha = 0{,}75 \Rightarrow \alpha = 37^\circ$ | 37° |
| | ***c*** | $F^2 = 25 - 9 = 16 \Rightarrow F = \pm 4$ | ±4 |
| | ***d*** | $\beta = \alpha$ | |



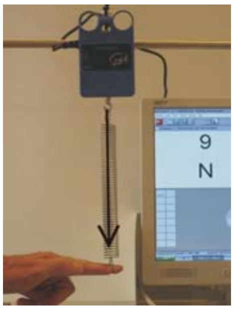

$\sin\alpha \left(= \frac{o}{s}\right) = \frac{14}{32} = 0{,}4375 \Rightarrow \alpha = \sin^{-1} 0{,}4375 = 25{,}9.. = 26^\circ$ 26°

$z^2 + 14^2 = 32^2 \Rightarrow z = \sqrt{32^2 - 14^2} = \sqrt{828} = 28{,}7.. = 29 \text{ cm}$ 29 cm

$\frac{x}{z} = \sin\alpha \Rightarrow x = z \cdot \sin\alpha = 28{,}7.. \cdot \sin 25{,}9.. = 12{,}5.. = 13 \text{ cm}$ 13 cm

$x^2 + y^2 = z^2 \Rightarrow y = \sqrt{z^2 - x^2} = \sqrt{28{,}7..^2 - 12{,}5..^2} = \sqrt{669{,}5..} = 25{,}8.. = 26 \text{ cm}$ 26 cm

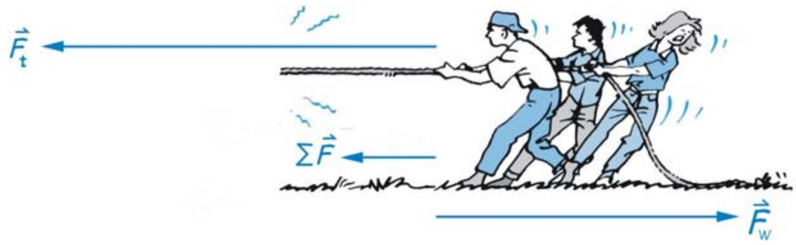
**1** De pijl voor $\vec{F}$ is weggevallen. Die had 1,8 cm lang moeten zijn.




–

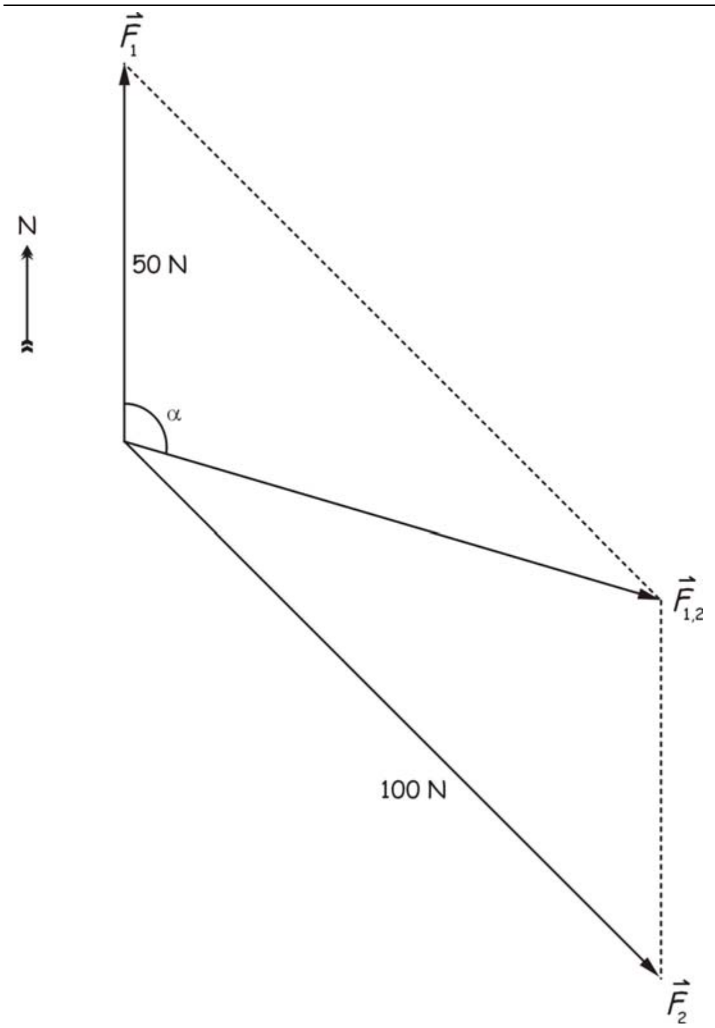
De pijl is 1,8 cm lang $\Rightarrow$ 1 cm $\triangleq$ 5 N

**2** ***a*** ***b*** ***c***




–

**3**




In figuur opmeten: α = 106° (hoek met noordrichting);
Resultante = 7,4 cm, staat voor 74 N

106°
74 N

**4** ***a***

$$\frac{F_x}{F} = \cos\alpha \Rightarrow F_x = F \cdot \cos\alpha = 35 \cdot \cos 56 = 19{,}5.. = 20 \text{ N}$$

$$\frac{F_y}{F} = \sin\alpha \Rightarrow F_y = F \cdot \sin\alpha = 35 \cdot \sin 56 = 29{,}0.. = 29 \text{ N}$$

20 N
29 N

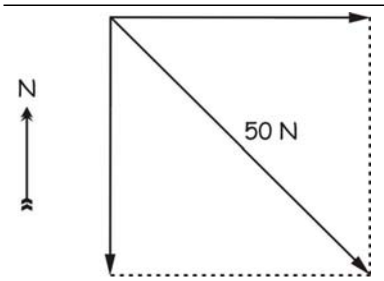
***b***

$$F_x{}^2 + F_y{}^2 = F^2 \Rightarrow F_y = \sqrt{F^2 - F_x{}^2} = \sqrt{47^2 - 32^2} = 34{,}4.. = 34 \text{ N}$$

$$\cos\alpha = \frac{F_x}{F} = \frac{32}{47} \Rightarrow \alpha = 47{,}0.. = 47^\circ$$

34 N
47°

**5** ***a***




–

***b***

$$\frac{F_Z}{F_{ZO}} = \cos 45 \Rightarrow F_Z = F_{ZO} \cdot \cos 45 = 50 \cdot \cos 45 = 35{,}3.. = 35 \text{ N}$$

Hetzelfde geldt voor $F_O$.

35 N

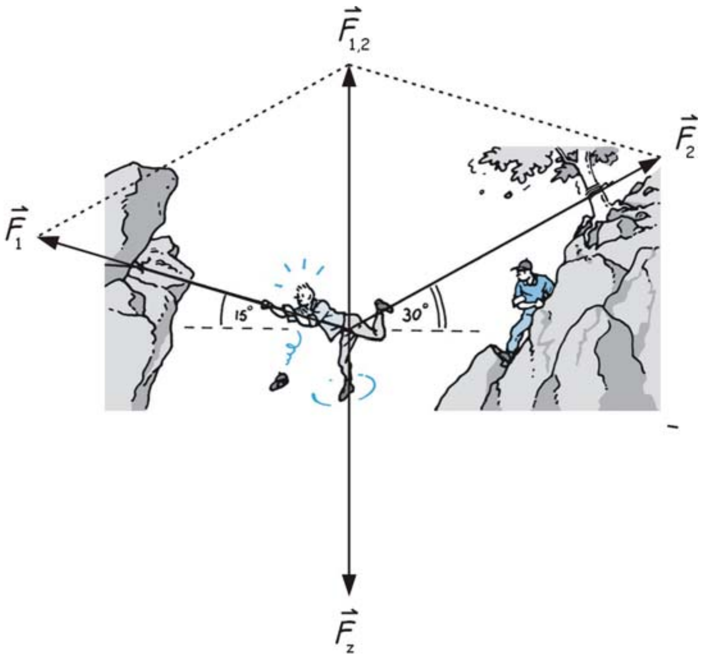
**6** ***a¹*** $F_x$ = 700·cos 20° = 658 N — 658 N

***a²*** $v$ is constant dus $F_w = F_x$ = 658 N — 658 N

| | | |
|---|---|---|
| ***b*** | $F_y$ = 700·sin 20º = 239,4.. = 239 N | 239 N |
| ***c***[1] | $F_z$ = 80·9,81 = 784,8.. = 785 N | 785 N |
| ***c***[2] | $F_n$ = $F_z$ – $F_y$ = 784,8 – 239,4 = 545 N | 545 N |

**Opgaven 4.2 – Krachten in evenwicht**

**7** ***a*** In de foute figuur is geen rekening gehouden met de tip op p. 88.




–

$\Sigma\vec{F}_{1,2}$ moet even groot zijn als $\vec{F}_z$ en tegengesteld daaraan gericht. In de gegeven constructie is de richting van de somkracht niet juist.

***b*** 1 cm $\hat{=}$ 200 N

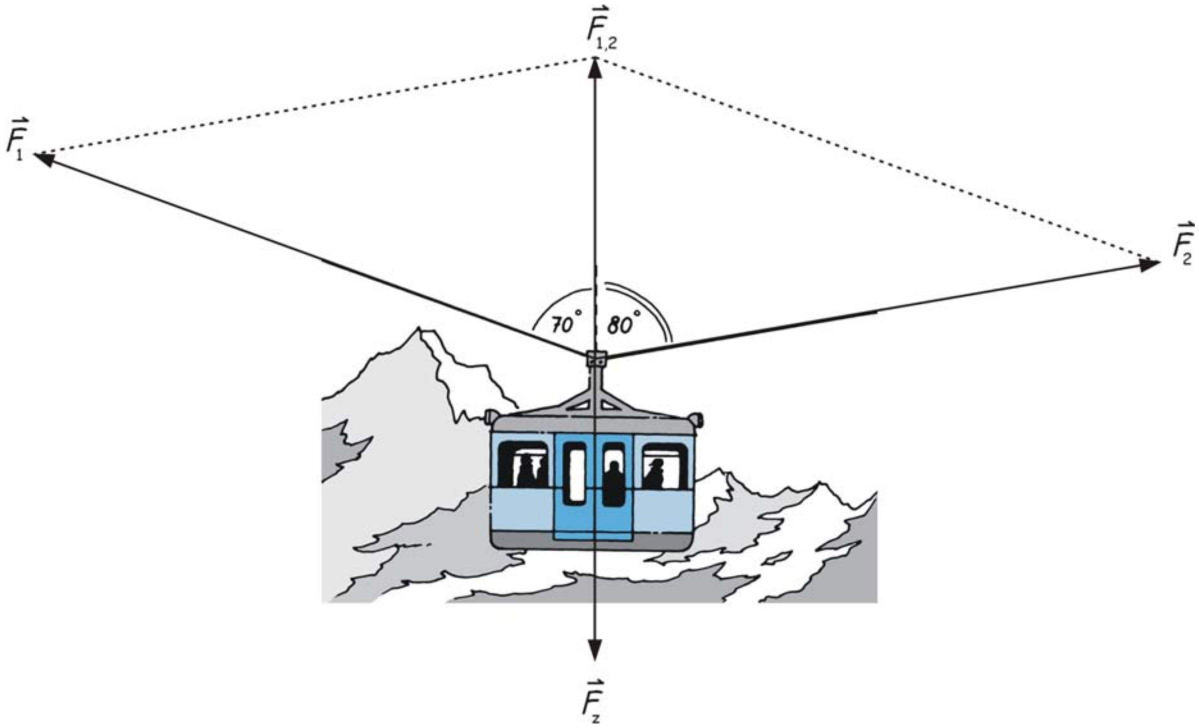
$F_1 = 4{,}3 \times 200 = 860 = 8{,}6 \cdot 10^2$ N — $8{,}6 \cdot 10^2$ N

$F_2 = 4{,}8 \times 200 = 960 = 9{,}6 \cdot 10^2$ N — $9{,}6 \cdot 10^2$ N

**8** ***a*** Het grootste deel van het gewicht wordt gedragen door de meest verticale draad. –

***b*** $2{,}0 \cdot 10^4$ N, want de cabine hangt dan bijna geheel aan de linkerkant van de kabel. $2{,}0 \cdot 10^4$ N

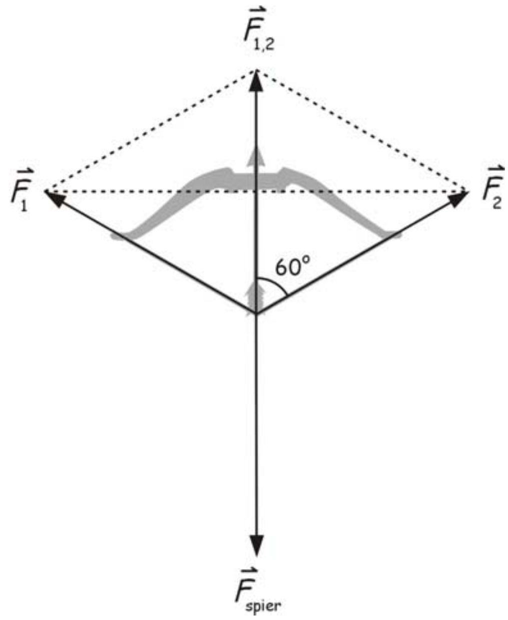
**c** 1 cm $\hat{=}$ $5 \cdot 10^3$ N




**d** $F_1 = 7{,}8 \times 5 \cdot 10^3 = 39 \cdot 10^3$ N — 39 kN

$F_2 = 7{,}6 \times 5 \cdot 10^3 = 38 \cdot 10^3$ N — 38 kN

**9 a**




–

$F_{span} = F_1 = F_2$ $F_{spier} = F_{1,2}$

**b** *1^e^ manier:*

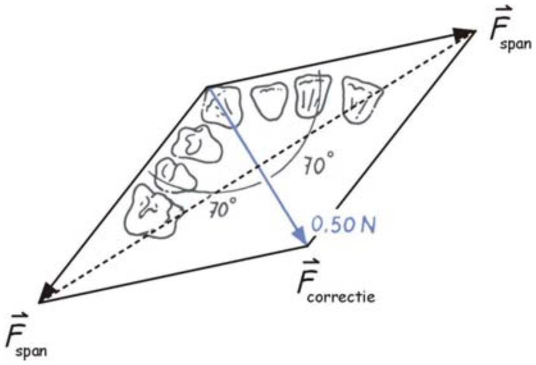
$F_{spier}$ is een van de zijden van een gelijkzijdige driehoek, waarvan één zijde al bekend is, nl. 130 N. Dus ook $F_{spier} = 130$ N

*2^e^ manier:* — 130 N

Het krachtenparallellogram is een ruit. Hierin is $F_{spier}$ een diagonaal, die door de andere diagonaal loodrecht door midden gedeeld wordt.

$F_{spier} = 2F_{span} \cdot \cos 60 = 2 \cdot 130 \cdot \cos 60 = 130$ N

**10**



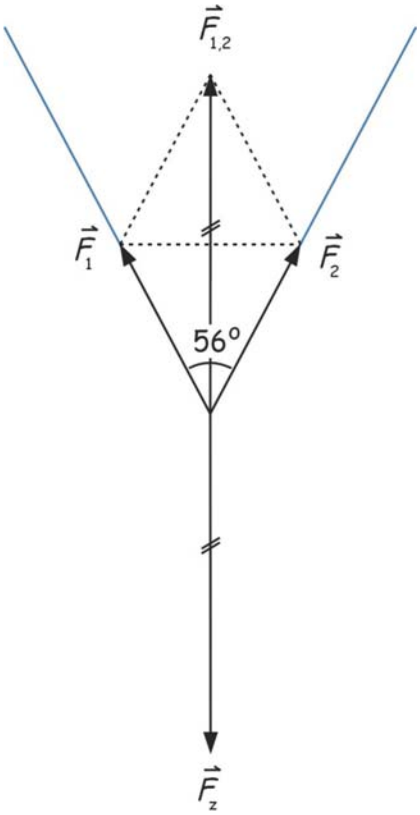

0,73 N

Het krachtenparallellogram is een ruit, dus

$$\frac{\frac{1}{2}F_{correctie}}{F_{span}} = \cos 70 \Rightarrow F_{span} = \frac{\frac{1}{2}F_{correctie}}{\cos 70} = \frac{0{,}25}{\cos 70} = 0{,}730.. = 0{,}73\ \text{N}$$

Dit kun je controleren in de constructie.

**11** ***a***




50 N

Opmeten in figuur: $F_1 = F_2 = 50$ N

***b*** Het krachtenparallellogram is een ruit, dus

$$\frac{\frac{1}{2}F_z}{F_{span}} = \cos 28 \Rightarrow F_{span} = \frac{\frac{1}{2}F_z}{\cos 28} = \frac{45}{\cos 28} = 50{,}9.. = 51\ \text{N}$$

51 N

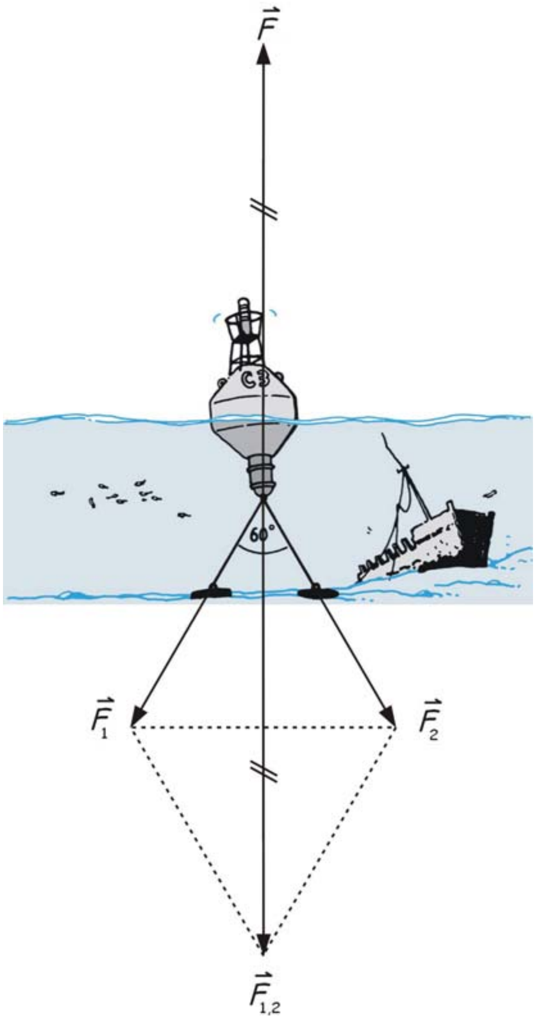
***c***

$$F_{span} < 70 \Rightarrow \cos\alpha = \frac{\frac{1}{2}F_z}{F_{span}} > \frac{45}{70} = 0{,}642.. \Rightarrow \alpha < \cos^{-1} 0{,}642.. = 49{,}99..$$

100°

De hoek tussen de touwen mag maximaal het dubbele zijn, dus 100°

**12** ***a*** De opwaartse kracht door het water is 4,4 kN; de zwaartekracht is 2,0 kN.
De spankrachten $\vec{F}_1$ en $\vec{F}_2$ in de kabels moeten dus samen de 2,4 kN omlaag leveren die nodig is om de boei op zijn plaats te houden.
De kracht $\vec{F}$ die verticaal omhoog wijst, is 2,4 kN groot.




1,4 kN

De kabels trekken verticaal omlaag met een kracht van 4,4 - 2,0 = 2,4 kN

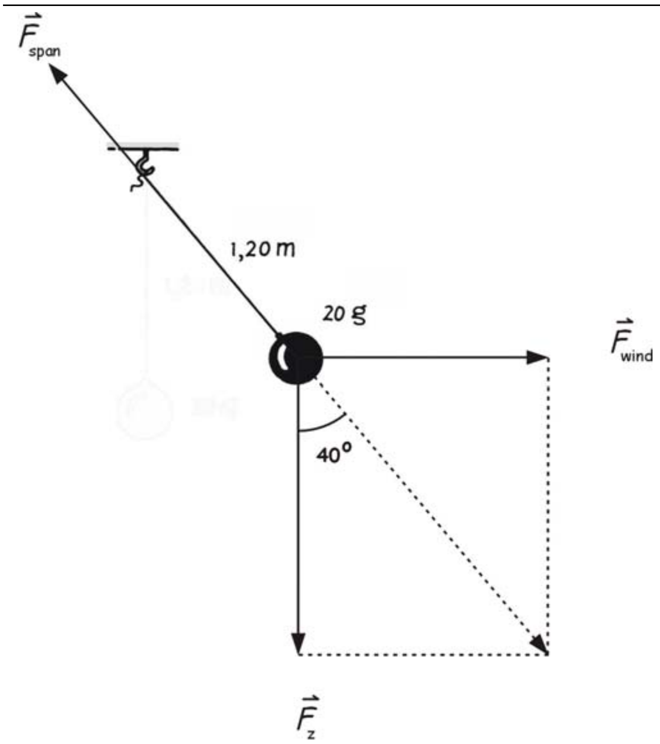
Het krachtenparallellogram is een ruit, dus

$$\frac{\frac{1}{2}F_{omlaag}}{F_{span}} = \cos 30 \Rightarrow F_{span} = \frac{\frac{1}{2}F_{omlaag}}{\cos 30} = \frac{1{,}2\ (\text{kN})}{\cos 30} = 1{,}38.. = 1{,}4\ \text{kN}$$

Dit vind je ook door opmeten in de figuur.

**13** ***a*** $F_z = m \cdot g = 0{,}020 \cdot 9{,}81 = 0{,}196..$ N = 0,20 N 0,20 N

***b***




***c***

$$\frac{F_{wind}}{F_z} = \tan 40 \Rightarrow F_{wind} = F_z \cdot \tan 40 = 0{,}196.. \cdot \tan 40 = 0{,}164.. = 0{,}16 \text{ N}$$

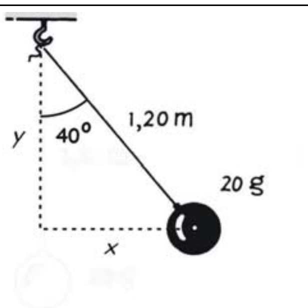
0,16 N

***d***

$$\frac{F_z}{F_{span}} = \cos 40 \Rightarrow F_{span} = \frac{F_z}{\cos 40} = \frac{0{,}196..}{\cos 40} = 0{,}256.. = 0{,}26 \text{ N}$$

0,26 N

***e***



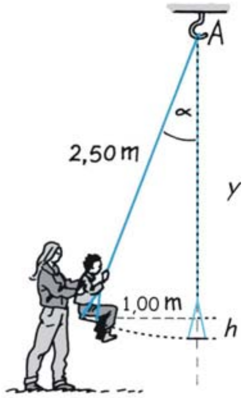
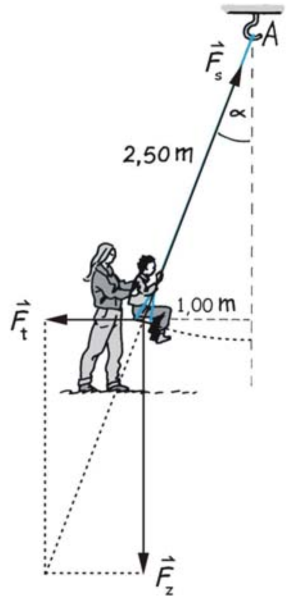

0,77 m

$$\frac{x}{1{,}20} = \sin 40 \Rightarrow x = 1{,}20 \cdot \sin 40 = 0{,}771.. = 0{,}77 \text{ m}$$

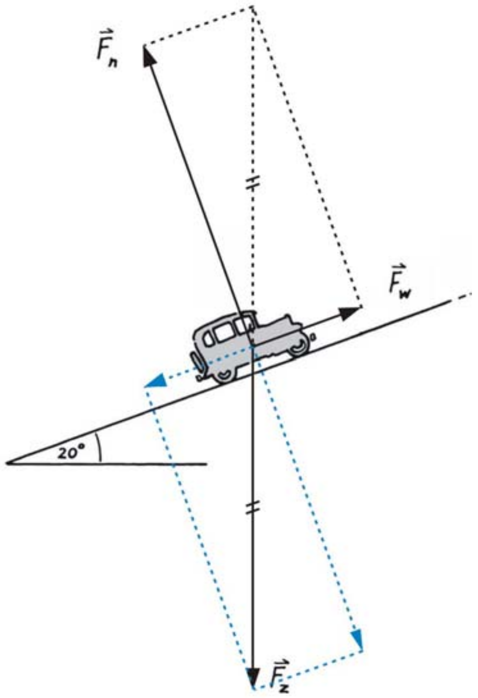
**14** ***a***

$$\sin\alpha = \frac{1{,}00}{2{,}50} \Rightarrow \alpha = 23{,}57.. = 24°$$

24°

| | | |
|---|---|---|
| ***b*** | <br>$y = \sqrt{2{,}50^2 - 1{,}00^2} = 2{,}291..\ \text{m}$<br>geeft $h = 2{,}50 - 2{,}291.. = 0{,}208.. = 0{,}21\ \text{m}$ | 0,21 m |
| ***c*** | $F_z$ = 17·9,81 = 167 N | 167 N |
| ***d*** | <br>1 cm ≙ 50 N | – |
| ***e*** | Meet $F_t$ is 1,3 cm lang ⇒ $F_t$ = 65 N | 65 N |
| ***f*** | $\tan\alpha = \dfrac{F}{F_z} \Rightarrow F = 17 \cdot 9{,}8 \cdot \tan 23{,}6^\circ = 73\ \text{N}$ | 73 N |

**15** ***a***
***b***

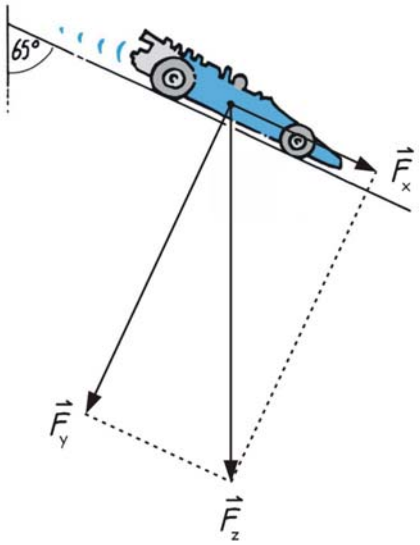



–

***c*** Zowel langs de helling als loodrecht op de heling geldt $\Sigma F = 0$
Langs de helling:
$F_w = F_{z,x} = F_z \cdot \sin 20 = 9{,}0 \cdot 10^3 \cdot \sin 20 = 3{,}07.. \cdot 10^3 = 3{,}1 \cdot 10^3$ N
Loodrecht op de helling:
$F_n = F_{z,y} = F_z \cdot \cos 20 = 9{,}0 \cdot 10^3 \cdot \cos 20 = 8{,}45.. \cdot 10^3 = 8{,}5 \cdot 10^3$ N

8,5 kN
3,1 kN

**16** ***a***




–

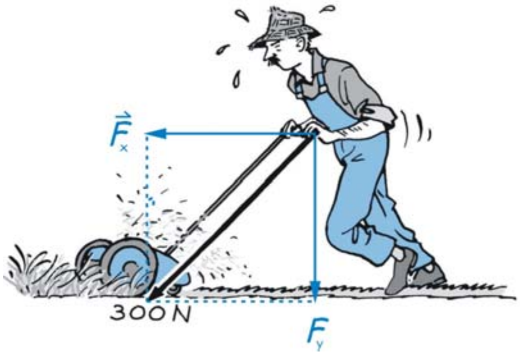
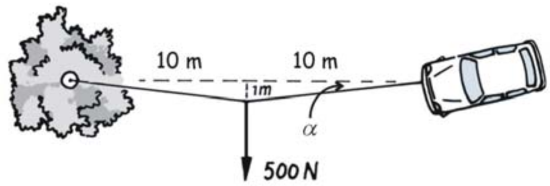
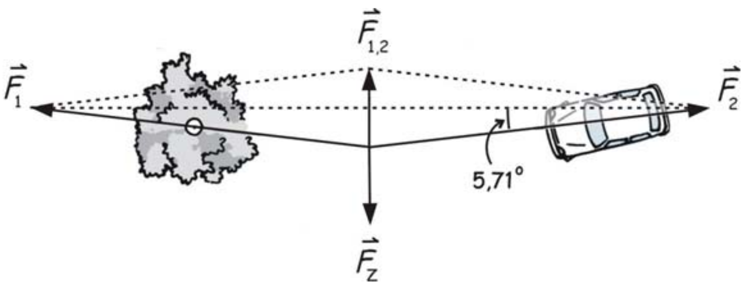
***b*** $F_x = F_z \cdot \cos\alpha = 1{,}00 \cdot \cos 65 = 0{,}422.. = 0{,}42$ N
$F_y = F_z \cdot \sin\alpha = 1{,}00 \cdot \sin 65 = 0{,}906.. = 0{,}91$ N

0,42 N
0,91 N

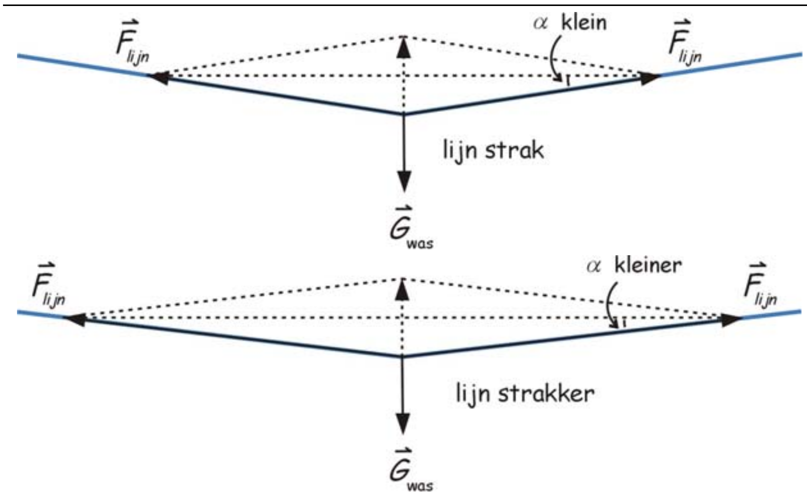
***c*** $F_z = m \cdot g \Rightarrow 1{,}00 = m \cdot 9{,}81 \Rightarrow m = \dfrac{1{,}00}{9{,}81} = 0{,}101..$ kg

$F_{z,x} = m \cdot a_x \Rightarrow 0{,}422.. = 0{,}101.. \cdot a_x \Rightarrow a_x = \dfrac{0{,}422..}{0{,}101..} = 4{,}14.. = 4{,}1$ m/s$^2$

4,1 m/s$^2$

| | | | |
|---|---|---|---|
| | | **Opgaven hoofdstuk 4** | |
| **17** | | Klopt, want $3^2 + 4^2 = 5^2$ | – |
| **18** | ***a*** | De pijl is 3,0 cm lang $\Rightarrow$ 1,0 cm $\hat{=}$ 100 N | – |
| | ***b*** <br> ***c*** |   | – |
| | ***d*** | $F_x$ is 2,2 cm lang $\Rightarrow$ $2{,}2 \cdot 10^2$ N en <br> $F_y$ os 2,0 cm lang $\Rightarrow$ $2{,}0 \cdot 10^2$ N | $2{,}2 \cdot 10^2$ N <br> $2{,}0 \cdot 10^2$ N |
| | ***e*** | $F_x$ = 300·cos43° = 219 N <br> $F_y$ = 300·sin43° = 205 N | 219 N <br> 205 N |
| **19** | |   <br> $\tan\alpha = \frac{1}{10} \Rightarrow \alpha = 5{,}71..$ <br>   <br> $\frac{250}{F_{auto}} = \sin 5{,}71.. \Rightarrow F_{auto} = \frac{250}{\sin 5{,}71..} = 2512,.. = 2{,}5 \cdot 10^3$ N | 2,5 kN |

**20** ***a***

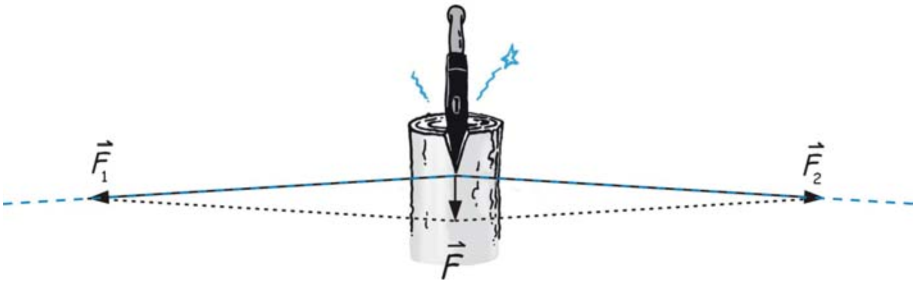



$$\frac{\frac{1}{2}G_{was}}{F_{lijn}} = \sin\alpha \rightarrow F_{lijn} = \frac{\frac{1}{2}G_{was}}{\sin\alpha}$$

Hoe strakker de draad gespannen is, hoe kleiner α en (sin α) en hoe groter $F_{lijn}$.

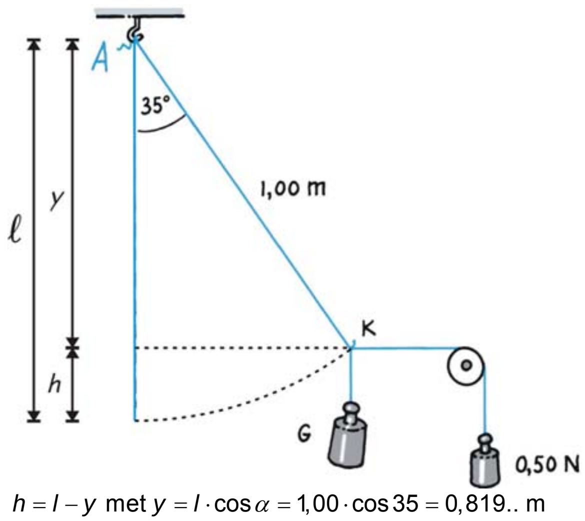
–

***b***




–

**21** ***a***

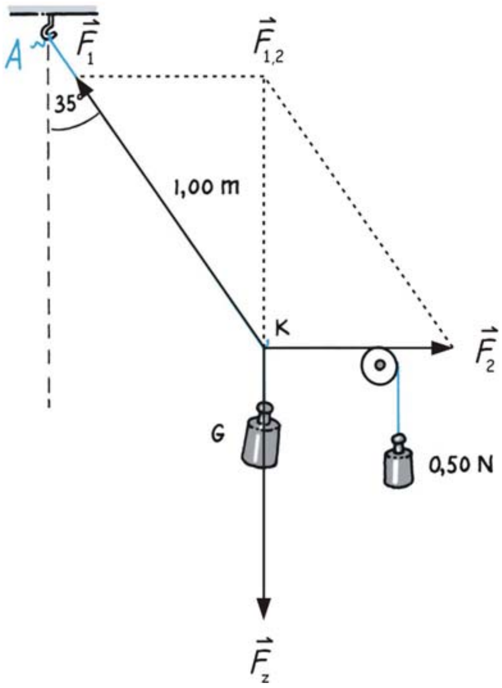



0,18 m

$h = l - y$ met $y = l \cdot \cos\alpha = 1{,}00 \cdot \cos 35 = 0{,}819..$ m

$\Rightarrow h = 1{,}00 - 0{,}819.. = 0{,}180.. = 0{,}18$ m

**b**




$\vec{F}_z = -\Sigma \vec{F}_{1,2}$

–

**c** $\dfrac{0,50}{F_{span}} = \sin 35 \Rightarrow F_{span} = \dfrac{0,50}{\sin 35} = 0,871.. = 0,87\ \text{N}$ — 0,87 N

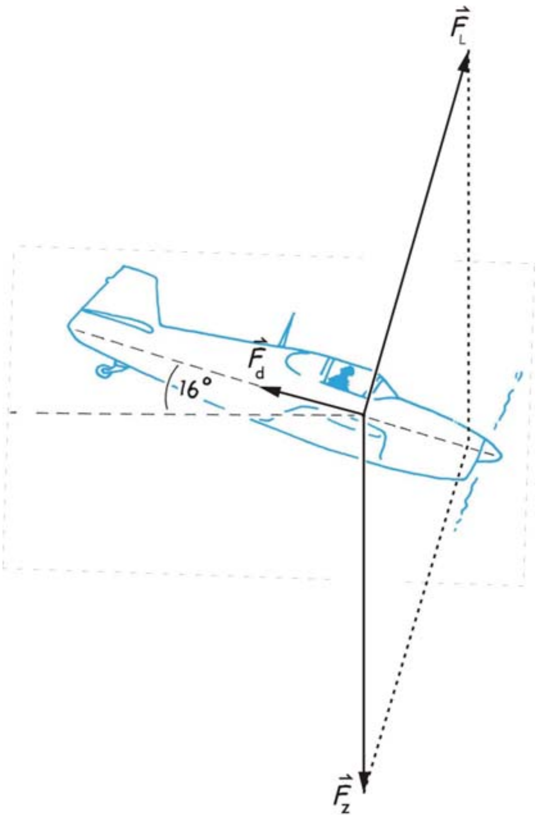
**d** $\dfrac{0,50}{G} = \tan 35 \Rightarrow G = \dfrac{0,50}{\tan 35} = 0,714.. = 0,71\ \text{N}$ — 0,71 N

**22 a** α = 62° — 62°

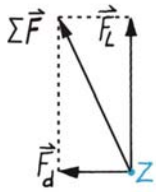
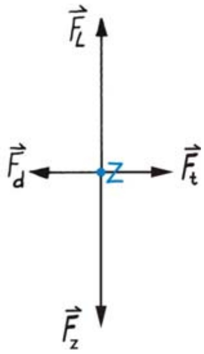
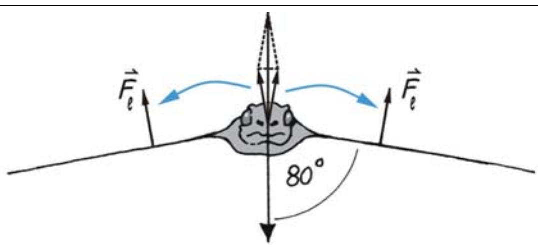
**b** $F_z = 0,500 \cdot 9,81 = 4,905 = 4,91$ N

$F_x = F_z \cdot \cos \alpha \Rightarrow F_x = 4,905 \cdot \cos 62° = 2,3$ N — 2,3 N

**23 a**



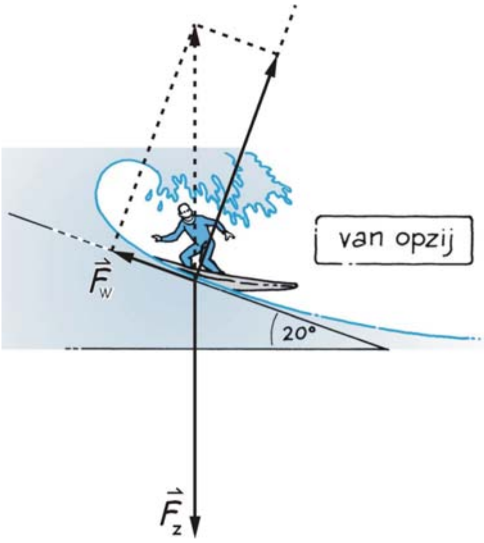

–

| | | | |
|---|---|---|---|
| | **b** | $f = \dfrac{F_L}{F_d} = \tan 74° = 3{,}5$ | 3,5 |
| | **c** | 3,5 km verder bij 1 km daling $\Rightarrow$ 35 km verder bij 10 km daling. | 35 km |
| **24** | **a** | Naar rechts, want $\vec{F}_d$ wijst naar links. | – |
| | **b** |  | – |
| | **c** | Met alleen deze twee wijst de resultante schuin omhoog naar achteren!<br>De zwaartekracht wijst verticaal omlaag en de kracht van de motor wijst naar rechts. | – |
| | **d** | $F_L = F_z = 395 \cdot 10^3 \cdot 9{,}81 = 3{,}9 \cdot 10^6$ N | $3{,}9 \cdot 10^6$ N |
| | **e** | <br>$F_d$ is 10 mm lang en $F_L$ is 20 mm lang $\Rightarrow$ $F_d = 1{,}9 \cdot 10^6$ N $F_t = F_d$ | $1{,}9 \cdot 10^6$ N |
| **25** | **a** | $x = v \cdot t \Rightarrow t = \dfrac{15}{3{,}5} = 4{,}3$ s | 4,3 s |
| | **b** | Dan zou $\vec{F}_d$ zorgen voor vertraging, terwijl er staat dat de vaart constant is. | – |
| | **c** | <br>$\sin 80° = \dfrac{F_L}{\frac{1}{2} \cdot F_z}$ en $F_z = 0{,}150 \cdot 9{,}81 = 1{,}47$ N $\Rightarrow$ $F_L = ½ \cdot 1{,}47 \cdot \sin 80° = 0{,}72$ N | 0,72 N |

## Toets

**1** **Surfen**

| | | |
|---|---|---|
| *a* | $\cos 50° = \frac{5{,}0}{v} \Rightarrow v = 7{,}8$ m/s | 7,8 m/s |
| *b* | $\Sigma\vec{F} = \vec{0}$ want de vaart is constant | – |
| *c* | $\tan 50° = \frac{40}{y} \Rightarrow y = 34$ m | 34 m |
| *d* | Neem aan dat we dwars op de plank kijken. | $2{,}9 \cdot 10^2$ N |



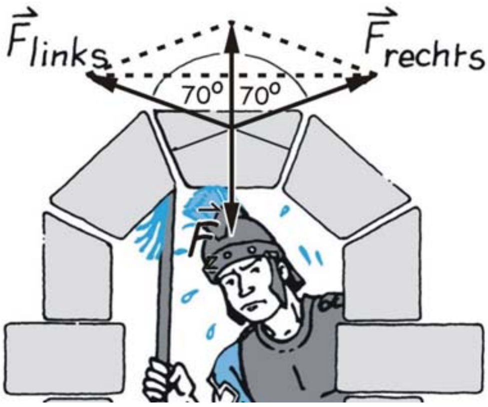

$\sin 20° = \frac{F_w}{F_z}$ en $F_z = 85 \cdot 9{,}81 = 8{,}3 \cdot 10^2$ N

$F_w = 8{,}3 \cdot 10° \cdot \sin 20° = 2{,}9 \cdot 10^2$ N

**2** **Een Romeinse boog**

| | | |
|---|---|---|
| *a* | De pijlen zijn 20 mm lang $\Rightarrow$ $F = \frac{20}{5} \cdot 1 \cdot 10^3 = 4 \cdot 10^3$ N | $3 \cdot 10^3$ N |

**b**




2,8·10² kg

$\cos 70° = \dfrac{\frac{1}{2}F_z}{4 \cdot 10^3} \Rightarrow F_z = 2 \cdot 4 \cdot 10^3 \cdot \cos 70° = 2{,}74 \cdot 10^3 \text{ N} \Rightarrow$

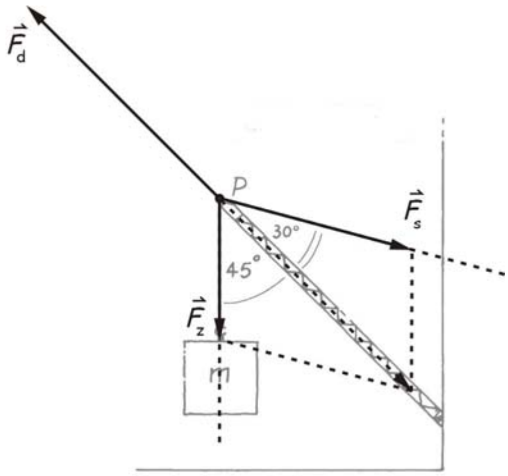
$m = \dfrac{2{,}74 \cdot 10^3}{9{,}81} = 2{,}8 \cdot 10^2 \text{ kg}$

**c** Dan zijn de krachten kleiner want ze werken dan beter samen i.p.v. tegen elkaar. –

**3** **Een hijskraan**

***a*¹** $\vec{F}_d$ is in het boek 1,8 cm lang ⇒ $F_d = 4{,}5 \cdot 10^3$ N

***a*²** In de volgende figuur is hij 2x zo groot.

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***b*¹** Opmeten geeft: $F_z = 2{,}4 \cdot 10^3$ N — 2,4·10² N

***b*²** $F_z = m \cdot g \Rightarrow m = 2{,}4 \cdot 10^2$ kg — 2,4·10² kg